

One-loop stau masses in the effective potential approach

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Abstract. We calculate the one-loop contributions to the tau slepton masses in the Minimal Supersymmetric Standard Model in the effective potential approach. For the majority of parameter space under study, those corrections are shown to elevate the value of the lightest stau mass.

Recently a full one-loop charge breaking effective potential was calculated for the case where, other than H_1^0 and H_2^0 , the fields τ_L and τ_R also acquire non-zero vevs [1], l and τ respectively. Its minimisation, and impact thereof on CCB bounds, was undertaken in ref. [2]. In this article we will not worry about CCB, instead simply note that the full dependence on l and τ in the effective potential enables us to calculate the one-loop stau masses - they are approximated by the second derivatives of the effective potential [3], a procedure that has been shown to give very accurate results [4], at least for the Higgs masses. Recently this approach has been used to calculate two-loop corrections to the CP-even Higgs boson masses [5]. Following the conventions and notation of ref. [1], we recall that we consider the MSSM with Yukawa couplings set to zero for the first and second generations, the superpotential of the model being given by

$$W = \lambda_t H_2 Q t_R + \lambda_b H_1 Q b_R + \lambda_\tau H_1 L \tau_R + \mu H_2 H_1 \quad . \quad (1)$$

Supersymmetry is softly broken by adding to the potential explicit mass terms for the gauginos and scalar partners, and bilinear and trilinear terms similar in form to those present in the superpotential above, but multiplied by coefficients B and A_i . When the fields H_1^0 , H_2^0 , τ_L and τ_R have vevs $v_1/\sqrt{2}$, $v_2/\sqrt{2}$, $l/\sqrt{2}$ and $\tau/\sqrt{2}$ respectively, the tree-level potential is given by

$$V_0 = \frac{\lambda_\tau^2}{4} [v_1^2 (l^2 + \tau^2) + l^2 \tau^2] - \frac{\lambda_\tau}{\sqrt{2}} (A_\tau v_1 + \mu v_2) l \tau + \frac{1}{2} (m_1^2 v_1^2 + m_2^2 v_2^2 + m_L^2 l^2 + m_\tau^2 \tau^2) - B \mu v_1 v_2 + \frac{g^2}{32} (v_2^2 - v_1^2 - l^2 + 2 \tau^2)^2 + \frac{g_2^2}{32} (v_2^2 - v_1^2 + l^2)^2 \quad . \quad (2)$$

The one-loop contributions to the effective potential are given, as usual, by

$$\Delta V_1 = \sum_\alpha \frac{n_\alpha}{64\pi^2} M_\alpha^4 \left(\log \frac{M_\alpha^2}{M^2} - \frac{3}{2} \right) \quad , \quad (3)$$

where the M_α are the tree-level masses of particles of spin s_α , M is the renormalisation scale and $n_\alpha = (-1)^{2s_\alpha} (2s_\alpha + 1) C_\alpha Q_\alpha$, with C_α the number of colour degrees of freedom and Q_α

counting the number of particle-antiparticle states. The presence in the potential of vevs carrying electric charge complicates the mass matrices considerably, by causing mixing between neutral and charged fields. For example, the tau sneutrinos are mixed with the charged higgs fields, originating a three by three mass matrix [1].

1 The effective potential approach

The one-loop contributions to the stau masses in the effective potential approach (e.p.a.) will be given by the second derivatives of ΔV_1 with respect to l and τ , so that we have

$$[m_\tau^2]_{ij} \simeq \frac{\partial^2 V_0}{\partial v_i \partial v_j} + \sum_\alpha \frac{n_\alpha}{32\pi^2} M_\alpha^2 \frac{\partial^2 M_\alpha^2}{\partial v_i \partial v_j} \left(\log \frac{M_\alpha^2}{M^2} - \frac{3}{2} \right) \Big|_{l=\tau=0}, \quad (4)$$

with $\{v_i, v_j\} = \{l, \tau\}$, and we have used the fact that

$$\frac{\partial M_\alpha^2}{\partial l} \Big|_{l=\tau=0} = \frac{\partial M_\alpha^2}{\partial \tau} \Big|_{l=\tau=0} = 0, \quad (5)$$

as follows trivially from analysing the $\{l, \tau\}$ dependence of the tree-level potential (2). With mass matrices as large as six by six, it would seem impossible to find analytical expressions for (4). We can however apply the same trick that allowed us to compute the derivatives $\partial M_\alpha^2 / \partial v_i$ in ref. [2], to wit: the squared masses M_α^2 are the solutions λ of the eigenvalue equation $F = \det([M_\alpha^2] - \lambda \mathbb{1}) = 0$. This equation - a simple polynomial of degree n for an $n \times n$ mass matrix - implicitly determines the λ in terms of the vevs present in the theory. Using (5) as well as the implicit function theorem we obtain

$$\frac{\partial^2 M_\alpha^2}{\partial v_i \partial v_j} = - \frac{\frac{\partial^2 F}{\partial v_i \partial v_j}}{\frac{\partial F}{\partial \lambda}} \Big|_{\lambda=M_\alpha^2}. \quad (6)$$

The derivatives of the determinant F are very easy to compute - taking into account that F is of the form

$$F = \sum_{i,j,k,\dots} a_{1i} a_{2j} a_{3k} \dots \epsilon_{ijk\dots}, \quad (7)$$

we get

$$\frac{\partial F}{\partial x} = \sum_{i,j,k,\dots} \left(\frac{\partial a_{1i}}{\partial x} a_{2j} a_{3k} \dots + a_{1i} \frac{\partial a_{2j}}{\partial x} a_{3k} \dots + a_{1i} a_{2j} \frac{\partial a_{3k}}{\partial x} \dots + \dots \right) \epsilon_{ijk\dots}, \quad (8)$$

which is to say, the derivative of the determinant of an $n \times n$ matrix becomes the sum of the determinants of n matrices, each identical to the initial matrix except for a line replaced with derivatives of the original coefficients. Along similar lines we see that the second derivatives of F would originate n^2 determinants. The final expressions obtained in this way for (4) are quite involved and we present them in the next section as a function of the sparticles' masses - these can be computed either analytically (the only masses that are more involved are the neutralino's; see, for instance, ref. [6]) or numerically. At this point we must recall that the e.p.a. gives only an approximation to the physical masses - in fact, from general principles one sees that [3, 4]

$$[m_\tau^2]_{ij}^{e.p.a.} = \frac{\partial^2 V^{eff}}{\partial v_i \partial v_j} = -\Gamma_{ij}(p^2 = 0), \quad (9)$$

where the $\Gamma_{ij}(p^2)$ are the inverse propagators of the stau sector, given by ¹

$$\Gamma_{ij}(p^2) = p^2 \delta_{ij} - M_{ij}^2 + \hat{\Pi}_{ij}(p^2) \quad . \quad (10)$$

The matrix M_{ij}^2 stands for the coefficients of the terms in the effective potential which are quadratic in the stau fields, and is given by

$$M_{ij}^2 = \begin{pmatrix} m_L^2 + \frac{1}{2} [2(m_W^2)' - (m_Z^2)'] \cos(2\beta) & \bar{m}_\tau' (A_\tau - \mu \cot \beta) \\ \bar{m}_\tau' (A_\tau - \mu \cot \beta) & m_\tau^2 + [(m_W^2)' - (m_Z^2)'] \cos(2\beta) \end{pmatrix} \quad , \quad (11)$$

where \bar{m}_τ represents the tau fermion mass. The parameters in this mass matrix are all one-loop renormalised. The primes in the masses indicate that these are the masses in the e.p.a. approximation, that is, $(m_W^2)' = g_2^2(v_1^2 + v_2^2)/4$, $(m_Z^2)' = (g'^2 + g_2^2)(v_1^2 + v_2^2)/4$ and $\bar{m}_\tau' = \lambda_\tau v_1/\sqrt{2}$. These are related to the physical, “unprimed” masses by

$$\begin{aligned} (m_W^2)' &= m_W^2 + \hat{\Pi}_{WW}(m_W^2) \quad , \\ (m_Z^2)' &= m_Z^2 + \hat{\Pi}_{ZZ}(m_Z^2) \quad , \\ (m_\tau)' &= m_\tau + \hat{\Pi}_{\tau\tau}(m_\tau) \quad . \end{aligned} \quad (12)$$

Our input parameters are the experimental values of the masses, so we express M_{ij}^2 in terms of unprimed quantities, so that $M_{ij}^2 = \bar{M}_{ij}^2 + \bar{\Pi}_{ij}$ - the matrix \bar{M}_{ij}^2 is identical in form to eq. (11), except the “primed” masses are replaced by the physical ones, and we have

$$\bar{\Pi}_{ij} = \begin{pmatrix} \frac{1}{2} [2\hat{\Pi}_{WW}(m_W^2) - \hat{\Pi}_{ZZ}(m_Z^2)] \cos(2\beta) & \hat{\Pi}_{\tau\tau}(m_\tau) (A_\tau - \mu \cot \beta) \\ \hat{\Pi}_{\tau\tau}(m_\tau) (A_\tau - \mu \cot \beta) & m_\tau^2 + [\hat{\Pi}_{ZZ}(m_Z^2) - \hat{\Pi}_{WW}(m_W^2)] \cos(2\beta) \end{pmatrix} \quad . \quad (13)$$

So, we can rewrite the inverse propagators of eq. (10) as

$$\Gamma_{ij}(p^2) = p^2 \delta_{ij} - \bar{M}_{ij}^2 + \hat{\Pi}_{ij}(0) + \Delta_{ij}(p^2) \quad , \quad (14)$$

where we define $\Delta_{ij}(p^2)$ as

$$\Delta_{ij}(p^2) = \hat{\Pi}_{ij}(p^2) - \bar{\Pi}_{ij} - \hat{\Pi}_{ij}(0) \quad . \quad (15)$$

Thus, the e.p.a. expressed by eq. (9) consists of neglecting the quantity $\Delta_{ij}(p^2)$ - which is equivalent to calculating the stau self energies $\hat{\Pi}_{ij}$ at zero external momentum and neglecting the effects of the gauge boson and tau self energies present in $\bar{\Pi}_{ij}$. The careful comparison between the diagrammatic and the effective potential approaches of ref. [4] reached similar conclusions for the higgs' masses. It was shown there that the e.p.a. produces extremely good results, the masses thus calculated differing from the “real” masses by very small amounts, at most ~ 3 GeV. It is reasonable to expect, then, that the e.p.a.-calculated stau masses will be comparably accurate. We can also expect the one-loop mass corrections to be small - the second derivatives of the masses with order to $\{l, \tau\}$ will always produce coefficients multiplied by the smaller couplings, g' , g_2 and λ_τ .

¹We follow the conventions of ref. [4] and absorb a factor of i in the definition of Γ ; furthermore, the $\hat{\Pi}$ are understood as being the real part of the one-loop renormalised self-energies.

2 One-loop stau masses

We now list all one-loop contributions to the stau mass matrix as per eq. (4). The sparticle masses are well known in the literature and the factors n_α are given in ref. [1]², so we list only the non-zero second derivatives of the masses. We adopt the convention $X_{v_i v_j}$ to denote the second derivative of the quantity X with respect to $\{v_i, v_j\} = \{l, \tau\}$ (and recall that these derivatives are evaluated at $l = \tau = 0$). The simplest of these contributions are those of the first and second generation squarks and sleptons and the gauge bosons, given by

$$\begin{aligned}
M_{\tilde{u}_{1,l}}^2 &= -\frac{1}{12}(g'^2 + 3g_2^2) & M_{\tilde{u}_{1,\tau\tau}}^2 &= \frac{1}{6}g'^2 & M_{\tilde{u}_{2,l}}^2 &= \frac{1}{3}g'^2 & (16) \\
M_{\tilde{u}_{2,\tau\tau}}^2 &= -\frac{2}{3}g'^2 & M_{\tilde{d}_{1,l}}^2 &= \frac{1}{12}(3g_2^2 - g'^2) & M_{\tilde{d}_{1,\tau\tau}}^2 &= \frac{1}{6}g'^2 \\
M_{\tilde{d}_{2,l}}^2 &= -\frac{1}{6}g'^2 & M_{\tilde{d}_{2,\tau\tau}}^2 &= \frac{1}{3}g'^2 & M_{\tilde{e}_{1,l}}^2 &= \frac{1}{4}(g_2^2 + g'^2) \\
M_{\tilde{e}_{1,\tau\tau}}^2 &= -\frac{1}{2}g'^2 & M_{\tilde{e}_{2,l}}^2 &= -\frac{1}{2}g'^2 & M_{\tilde{e}_{2,\tau\tau}}^2 &= g'^2 \\
M_{\tilde{\nu}_e,l}^2 &= \frac{1}{4}(g'^2 - g_2^2) & M_{\tilde{\nu}_e,\tau\tau}^2 &= -\frac{1}{2}g'^2 & M_{\tilde{W},l}^2 &= \frac{1}{2}g_2^2 \\
M_{\tilde{Z},l}^2 &= \frac{1}{2}(g_2^2 + g'^2) \cos^2(2\theta_W) & & & & & (17)
\end{aligned}$$

For the stop, we have

$$\begin{aligned}
M_{\tilde{t}_{1,l}}^2 &= \frac{3(g'^2 - g_2^2) M_{\tilde{t}_1}^2 - 4g'^2 a_{\tilde{t}} + (g'^2 + 3g_2^2) c_{\tilde{t}}}{12(M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2)} \\
M_{\tilde{t}_{1,\tau\tau}}^2 &= -\frac{g'^2}{6} \frac{3M_{\tilde{t}_1}^2 - 4a_{\tilde{t}} + c_{\tilde{t}}}{M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2} , & (18)
\end{aligned}$$

with

$$\begin{aligned}
a_{\tilde{t}} &= m_Q^2 + \frac{1}{2}\lambda_t^2 v_2^2 + \frac{1}{24}(g'^2 - 3g_2^2)(v_2^2 - v_1^2) \\
c_{\tilde{t}} &= m_t^2 + \frac{1}{2}\lambda_t^2 v_2^2 - \frac{g'^2}{6}(v_2^2 - v_1^2) , & (19)
\end{aligned}$$

and identical expressions for the second stop, with the substitution $M_{\tilde{t}_1}^2 \leftrightarrow M_{\tilde{t}_2}^2$. For the sbottom we have

$$\begin{aligned}
M_{\tilde{b}_{1,l}}^2 &= \frac{3(g_2^2 - g'^2) M_{\tilde{b}_1}^2 + 2g'^2 a_{\tilde{b}} + (g'^2 - 3g_2^2) c_{\tilde{b}}}{12(M_{\tilde{b}_1}^2 - M_{\tilde{b}_2}^2)} \\
M_{\tilde{b}_{1,\tau\tau}}^2 &= \frac{g'^2}{6} \frac{3M_{\tilde{b}_1}^2 - 2a_{\tilde{b}} - c_{\tilde{b}}}{M_{\tilde{b}_1}^2 - M_{\tilde{b}_2}^2} , & (20)
\end{aligned}$$

with

$$\begin{aligned}
a_{\tilde{b}} &= m_Q^2 + \frac{1}{2}\lambda_b^2 v_1^2 + \frac{1}{24}(g'^2 + 3g_2^2)(v_2^2 - v_1^2) \\
c_{\tilde{b}} &= m_b^2 + \frac{1}{2}\lambda_b^2 v_1^2 + \frac{g'^2}{12}(v_2^2 - v_1^2) . & (21)
\end{aligned}$$

²We take this opportunity to correct a misprint in ref. [1], though: the factor n_α for the first and second generation sneutrinos is 4, counting two generations and the existence of both neutrinos and anti-neutrinos.

Again, identical expressions for the second sbottom with trivial substitutions. For the charginos we find

$$M_{\chi_1^\pm, ll}^2 = g_2^2 \frac{\mu^2 + \frac{1}{2} g_2^2 v_1^2 - M_{\chi_1^\pm}^2}{M_{\chi_2^\pm}^2 - M_{\chi_1^\pm}^2} \quad M_{\chi_1^\pm, \tau\tau}^2 = \lambda_\tau^2 \frac{M_2^2 + \frac{1}{2} g_2^2 v_2^2 - M_{\chi_1^\pm}^2}{M_{\chi_2^\pm}^2 - M_{\chi_1^\pm}^2}$$

$$M_{\chi_1^\pm, l\tau}^2 = \frac{\lambda_\tau g_2^2}{\sqrt{2}} \frac{\mu v_2 + M_2 v_1}{M_{\chi_2^\pm}^2 - M_{\chi_1^\pm}^2} . \quad (22)$$

For the neutralinos and tau lepton the expressions are more complex. From the formulae of ref. [2] it is easy to find ³

$$M_{\chi_i^0, xy} = - \frac{-B_{\chi^0, xy} M_{\chi_i^0}^4 - C_{\chi^0, xy} M_{\chi_i^0}^3 + D_{\chi^0, xy} M_{\chi_i^0}^2 + E_{\chi^0, xy} M_{\chi_i^0} + F_{\chi^0, xy}}{6 M_{\chi_i^0}^5 - 5 A_{\chi^0} M_{\chi_i^0}^4 - 4 B_{\chi^0} M_{\chi_i^0}^3 - 3 C_{\chi^0} M_{\chi_i^0}^2 + 2 D_{\chi^0} M_{\chi_i^0} + E_{\chi^0}} , \quad (23)$$

where the index i runs from 1 to 6, the two last entries being $\pm \lambda_\tau v_1 / \sqrt{2}$ (the tau mass) and the coefficients $A_{\chi^0}, \dots, F_{\chi^0}$ are given by

$$\begin{aligned} A_{\chi^0} &= M_1 + M_2 \\ B_{\chi^0} &= \frac{\lambda_\tau^2}{2} v_1^2 + \frac{1}{4} (g'^2 + g_2^2) (v_1^2 + v_2^2) + \mu^2 - M_1 M_2 \\ C_{\chi^0} &= -\frac{\lambda_\tau^2}{2} (M_1 + M_2) v_1^2 - \frac{1}{4} (g'^2 M_2 + g_2^2 M_1) (v_1^2 + v_2^2) + \frac{1}{2} (g'^2 + g_2^2) \mu v_1 v_2 - \\ &\quad \mu^2 (M_1 + M_2) \\ D_{\chi^0} &= \lambda_\tau^2 v_1^2 \left[\frac{1}{8} (g'^2 + g_2^2) (v_1^2 + v_2^2) - \frac{1}{2} M_1 M_2 + \frac{\mu^2}{2} \right] + \frac{1}{2} (g'^2 M_2 + g_2^2 M_1) \mu v_1 v_2 - \\ &\quad M_1 M_2 \mu^2 \\ E_{\chi^0} &= \lambda_\tau^2 v_1^2 \left[\frac{1}{4} (g'^2 + g_2^2) \mu v_1 v_2 - \frac{1}{8} (g'^2 M_2 + g_2^2 M_1) (v_2^2 - 2 v_1^2) - \frac{\mu^2}{2} (M_1 + M_2) \right] \end{aligned} \quad (24)$$

and their derivatives,

$$\begin{aligned} B_{\chi^0, ll} &= \frac{1}{2} (2 \lambda_\tau^2 + g'^2 + g_2^2) & B_{\chi^0, \tau\tau} &= (\lambda_\tau^2 + 2 g'^2) \\ C_{\chi^0, ll} &= -\frac{1}{2} \left[2 \lambda_\tau^2 (M_1 + M_2) + (g'^2 M_2 + g_2^2 M_1) \right] \\ C_{\chi^0, \tau\tau} &= -\left[\lambda_\tau^2 (M_1 + M_2) + 2 g'^2 M_2 \right] \\ C_{\chi^0, l\tau} &= -\frac{\lambda_\tau}{2\sqrt{2}} (3 g'^2 - g_2^2 - 2 \lambda_\tau^2) v_1 \\ D_{\chi^0, ll} &= \lambda_\tau^2 \left[\frac{1}{4} (g'^2 + g_2^2) (v_2^2 - 2 v_1^2) - M_1 M_2 \right] + \frac{\mu^2}{2} (g'^2 + g_2^2) \\ D_{\chi^0, \tau\tau} &= \lambda_\tau^2 \left[g'^2 v_1^2 + \frac{1}{4} (g'^2 + g_2^2) v_2^2 - M_1 M_2 \right] + \frac{1}{2} \left[g'^2 g_2^2 (v_1^2 + v_2^2) + 4 g'^2 \mu^2 \right] \\ D_{\chi^0, l\tau} &= \frac{\lambda_\tau^3}{\sqrt{2}} (M_1 + M_2) v_1 + \frac{\lambda_\tau}{2\sqrt{2}} \left[(g_2^2 M_1 - 3 g'^2 M_2) v_1 + (g'^2 - g_2^2) \mu v_2 \right] \\ E_{\chi^0, ll} &= \frac{\lambda_\tau^2}{4} \left[(g'^2 M_2 + g_2^2 M_1) (2 v_1^2 - v_2^2) - 2 (g'^2 + g_2^2) \mu v_1 v_2 \right] - \frac{\mu^2}{2} (g'^2 M_2 + g_2^2 M_1) \\ E_{\chi^0, \tau\tau} &= \frac{\lambda_\tau^2}{4} \left[4 g'^2 (\mu v_1 v_2 - M_2 v_1^2) - (g'^2 M_2 + g_2^2 M_1) v_2^2 \right] + g'^2 (g_2^2 v_1 v_2 - 2 \mu M_2) \mu \end{aligned}$$

³Notice that this is the second derivative of the mass, not its squared.

$$\begin{aligned}
E_{\chi^0, l\tau} &= \frac{\lambda_\tau^3}{4\sqrt{2}} \left[(g'^2 + g_2^2) v_2^2 - 4 M_1 M_2 \right] v_1 + \\
&\quad \frac{\lambda_\tau}{2\sqrt{2}} \left[\mu (g_2^2 M_1 - g'^2 M_2) v_2 - 2 g'^2 \mu^2 v_1 \right] \\
F_{\chi^0, ll} &= \frac{\lambda_\tau^2}{2} (g'^2 M_2 + g_2^2 M_1) \mu v_1 v_2 \\
F_{\chi^0, \tau\tau} &= -\lambda_\tau^2 g'^2 \mu M_2 v_1 v_2 \\
F_{\chi^0, l\tau} &= -\frac{\lambda_\tau^3}{4\sqrt{2}} (g'^2 M_2 + g_2^2 M_1) v_1 v_2^2 + \frac{\lambda_\tau}{\sqrt{2}} g'^2 \mu^2 M_2 v_1 \quad .
\end{aligned} \tag{25}$$

For the charged higgses,

$$\begin{aligned}
M_{H_1^\pm, ll}^2 &= -\frac{(2\lambda_\tau^2 - g_2^2)^2 v_1^2 (c_\pm - M_{H_1^\pm}^2) + g_2^2 v_2 \left[g_2^2 v_2 (a_\pm - M_{H_1^\pm}^2) + 2(2\lambda_\tau^2 - g_2^2) v_1 b_\pm \right]}{8(f_\pm - M_{H_1^\pm}^2)(a_\pm + c_\pm - 2M_{H_1^\pm}^2)} + \\
&\quad \frac{1}{4} (g'^2 + g_2^2) \frac{c_\pm - a_\pm}{a_\pm + c_\pm - 2M_{H_1^\pm}^2} \\
M_{H_1^\pm, \tau\tau}^2 &= -\lambda_\tau^2 \frac{A_\tau^2 (c_\pm - M_{H_1^\pm}^2) + \mu \left[\mu (a_\pm - M_{H_1^\pm}^2) - 2A_\tau b_\pm \right]}{(f_\pm - M_{H_1^\pm}^2)(a_\pm + c_\pm - 2M_{H_1^\pm}^2)} + \\
&\quad \frac{g'^2 (a_\pm - c_\pm) + 2\lambda_\tau^2 (c_\pm - M_{H_1^\pm}^2)}{2(a_\pm + c_\pm - 2M_{H_1^\pm}^2)} \\
M_{H_1^\pm, l\tau}^2 &= \lambda_\tau \frac{(2\lambda_\tau^2 - g_2^2) v_1 \left[\mu b_\pm - A_\tau (c_\pm - M_{H_1^\pm}^2) \right] + g_2^2 v_2 \left[\mu (a_\pm - M_{H_1^\pm}^2) - A_\tau b_\pm \right]}{2\sqrt{2}(f_\pm - M_{H_1^\pm}^2)(a_\pm + c_\pm - 2M_{H_1^\pm}^2)} \tag{26}
\end{aligned}$$

with coefficients

$$\begin{aligned}
a_\pm &= m_1^2 - \frac{g'^2}{8} (v_2^2 - v_1^2) + \frac{g_2^2}{8} (v_2^2 + v_1^2) & b_\pm &= B\mu + \frac{g_2^2}{4} v_1 v_2 \\
c_\pm &= m_2^2 + \frac{g'^2}{8} (v_2^2 - v_1^2) + \frac{g_2^2}{8} (v_2^2 + v_1^2) & f_\pm &= m_L^2 - \frac{1}{8} (g'^2 + g_2^2) (v_2^2 - v_1^2) \quad .
\end{aligned} \tag{27}$$

Similar expressions hold for $M_{H_2^\pm}^2$ - notice that if we perform a tree-level minimisation of the potential, this second eigenvalue is actually zero, reflecting the presence of a Goldstone boson. With a one-loop minimisation the tree-level charged higgs mass matrix produces two non-zero eigenvalues, albeit the second is quite small when compared to the first one, and therefore has a small impact in the one-loop potential. This second eigenvalue can even be negative [9] and its contribution to the potential neglected following the arguments of [10]. For the tau sneutrino we have

$$\begin{aligned}
M_{\tilde{\nu}_\tau, ll}^2 &= -\frac{(2\lambda_\tau^2 - g_2^2)^2 v_1^2 (c_\pm - M_{\tilde{\nu}_\tau}^2) + g_2^2 v_2 \left[g_2^2 v_2 (a_\pm - M_{\tilde{\nu}_\tau}^2) + 2(2\lambda_\tau^2 - g_2^2) v_1 b_\pm \right]}{8(a_\pm - M_{\tilde{\nu}_\tau}^2)(c_\pm - M_{\tilde{\nu}_\tau}^2) - 8b_\pm^2} + \\
&\quad \frac{1}{4} (g'^2 + g_2^2) \\
M_{\tilde{\nu}_\tau, \tau\tau}^2 &= \lambda_\tau^2 - \frac{1}{2} g'^2 - \lambda_\tau^2 \frac{A_\tau^2 (c_\pm - M_{\tilde{\nu}_\tau}^2) + \mu \left[\mu (a_\pm - M_{\tilde{\nu}_\tau}^2) - 2A_\tau b_\pm \right]}{(a_\pm - M_{\tilde{\nu}_\tau}^2)(c_\pm - M_{\tilde{\nu}_\tau}^2) - b_\pm^2} \\
M_{\tilde{\nu}_\tau, l\tau}^2 &= \lambda_\tau \frac{(2\lambda_\tau^2 - g_2^2) v_1 \left[\mu b_\pm - A_\tau (c_\pm - M_{\tilde{\nu}_\tau}^2) \right] + g_2^2 v_2 \left[\mu (a_\pm - M_{\tilde{\nu}_\tau}^2) - A_\tau b_\pm \right]}{2\sqrt{2} \left[(a_\pm - M_{\tilde{\nu}_\tau}^2)(c_\pm - M_{\tilde{\nu}_\tau}^2) - b_\pm^2 \right]} \quad .
\end{aligned} \tag{28}$$

The reason for the sharing of coefficients between the second derivatives of $M_{H^\pm}^2$ and $M_{\tilde{\nu}_\tau}^2$ is the mixing discussed in ref. [1]. For the pseudo-scalar Higgs the second derivatives are given by

$$\begin{aligned} M_{\tilde{H}_1, ll}^2 &= \frac{4\lambda_\tau^2 (c_{\tilde{H}} - M_{\tilde{H}_1}^2) + (g'^2 - g_2^2) (c_{\tilde{H}} - a_{\tilde{H}})}{4(M_{\tilde{H}_2}^2 - M_{\tilde{H}_1}^2)} + \frac{A_{ll}(M_{\tilde{H}_1}^2)}{D_{\tilde{H}}(M_{\tilde{H}_1}^2, M_{\tilde{H}_2}^2)} \\ M_{\tilde{H}_1, \tau\tau}^2 &= \frac{2\lambda_\tau^2 (c_{\tilde{H}} - M_{\tilde{H}_1}^2) - g'^2 (c_{\tilde{H}} - a_{\tilde{H}})}{2(M_{\tilde{H}_2}^2 - M_{\tilde{H}_1}^2)} + \frac{A_{\tau\tau}(M_{\tilde{H}_1}^2)}{D_{\tilde{H}}(M_{\tilde{H}_1}^2, M_{\tilde{H}_2}^2)} \\ M_{\tilde{H}_1, l\tau}^2 &= \frac{A_{l\tau}(M_{\tilde{H}_1}^2)}{D_{\tilde{H}}(M_{\tilde{H}_1}^2, M_{\tilde{H}_2}^2)} \end{aligned} \quad (29)$$

where the functions A are given by

$$\begin{aligned} A_{ll}(M_{\tilde{H}_1}^2) &= \lambda_\tau^2 (h_{\tilde{H}} - M_{\tilde{H}_1}^2) \left[2B\mu^2 A_\tau - \mu^2 (a_{\tilde{H}} - M_{\tilde{H}_1}^2) - A_\tau^2 (c_{\tilde{H}} - M_{\tilde{H}_1}^2) \right] \\ A_{\tau\tau}(M_{\tilde{H}_1}^2) &= \lambda_\tau^2 (j_{\tilde{H}} - M_{\tilde{H}_1}^2) \left[2B\mu^2 A_\tau - \mu^2 (a_{\tilde{H}} - M_{\tilde{H}_1}^2) - A_\tau^2 (c_{\tilde{H}} - M_{\tilde{H}_1}^2) \right] \\ A_{l\tau}(M_{\tilde{H}_1}^2) &= \lambda_\tau^2 i_{\tilde{H}} \left[\mu^2 (a_{\tilde{H}} - M_{\tilde{H}_1}^2) + A_\tau^2 (c_{\tilde{H}} - M_{\tilde{H}_1}^2) - 2B\mu^2 A_\tau \right] \end{aligned} \quad (30)$$

and the denominator $D_{\tilde{H}}$ by

$$D_{\tilde{H}}(M_{\tilde{H}_1}^2, M_{\tilde{H}_2}^2) = (M_{\tilde{H}_2}^2 - M_{\tilde{H}_1}^2) \left[(h_{\tilde{H}} - M_{\tilde{H}_1}^2) (j_{\tilde{H}} - M_{\tilde{H}_1}^2) - i_{\tilde{H}}^2 \right] \quad (31)$$

with coefficients

$$\begin{aligned} a_{\tilde{H}} &= m_1^2 - \frac{1}{8} (g'^2 + g_2^2) (v_2^2 - v_1^2) & c_{\tilde{H}} &= m_2^2 + \frac{1}{8} (g'^2 + g_2^2) (v_2^2 - v_1^2) \\ h_{\tilde{H}} &= m_L^2 + \frac{1}{8} (g_2^2 - g'^2) (v_2^2 - v_1^2) & i_{\tilde{H}} &= \frac{\lambda_\tau}{\sqrt{2}} (\mu v_2 - A_\tau v_1) \\ j_{\tilde{H}} &= m_\tau^2 + \frac{g'^2}{4} (v_2^2 - v_1^2) \end{aligned} \quad (32)$$

With the replacement $M_{\tilde{H}_1}^2 \leftrightarrow M_{\tilde{H}_2}^2$, we obtain the expressions for the second pseudo-scalar ⁴. For the Higgs scalars, we obtain

$$\begin{aligned} M_{h, ll}^2 &= \frac{4\lambda_\tau^2 (c_H - M_h^2) + (g'^2 - g_2^2) (c_H - a_H)}{4(M_H^2 - M_h^2)} + \frac{B_{ll}(M_h^2)}{D_H(M_h^2, M_H^2)} \\ M_{h, \tau\tau}^2 &= \frac{2\lambda_\tau^2 (c_H - M_h^2) + g'^2 (a_H - c_H)}{2(M_H^2 - M_h^2)} + \frac{B_{\tau\tau}(M_h^2)}{D_H(M_h^2, M_H^2)} \\ M_{h, l\tau}^2 &= \frac{B_{l\tau}(M_h^2)}{D_H(M_h^2, M_H^2)} \end{aligned} \quad (33)$$

where the denominator D_H has the expression

$$D_H(M_h^2, M_H^2) = (M_H^2 - M_h^2) \left[(h_H - M_h^2) (j_H - M_h^2) - i_H^2 \right] \quad (34)$$

and the functions B are given by

$$\begin{aligned} B_{ll}(M_h^2) &= 2(a_H - M_h^2) \left[2i_H f_{H,l} g_{H,l} - f_{H,l}^2 (j_H - M_h^2) - g_{H,l}^2 (h_H - M_h^2) \right] + \\ &\quad 2(c_H - M_h^2) \left[2i_H d_{H,l} e_{H,l} - d_{H,l}^2 (j_H - M_h^2) - e_{H,l}^2 (h_H - M_h^2) \right] + \end{aligned}$$

⁴Which, like in the case of the charged Higgses, is a Goldstone boson for a tree-level minimisation.

$$\begin{aligned}
& 4b_H [d_{H,l} f_{H,l} (j_{\bar{H}} - M_h^2) + e_{H,l} g_{H,l} (h_{\bar{H}} - M_h^2) - i_H (g_{H,l} d_{H,l} + e_{H,l} f_{H,l})] \\
B_{\tau\tau}(M_h^2) &= 2(a_H - M_h^2) [2i_H f_{H,\tau} g_{H,\tau} - f_{H,\tau}^2 (j_{\bar{H}} - M_h^2) - g_{H,\tau}^2 (h_{\bar{H}} - M_h^2)] + \\
& 2(c_H - M_h^2) [2i_H d_{H,\tau} e_{H,\tau} - d_{H,\tau}^2 (j_{\bar{H}} - M_h^2) - e_{H,\tau}^2 (h_{\bar{H}} - M_h^2)] + \\
& 4b_H [d_{H,\tau} f_{H,\tau} (j_{\bar{H}} - M_h^2) + e_{H,\tau} g_{H,\tau} (h_{\bar{H}} - M_h^2) - i_H (g_{H,\tau} d_{H,\tau} + e_{H,\tau} f_{H,\tau})] \\
B_{l\tau}(M_h^2) &= 2(a_H - M_h^2) [i_H (f_{H,l} g_{H,\tau} + f_{H,\tau} g_{H,l}) - f_{H,l} f_{H,\tau} (j_{\bar{H}} - M_h^2) - g_{H,l} g_{H,\tau} (h_{\bar{H}} - M_h^2)] + \\
& 2(c_H - M_h^2) [i_H (d_{H,l} e_{H,\tau} + d_{H,\tau} e_{H,l}) - d_{H,l} d_{H,\tau} (j_{\bar{H}} - M_h^2) - e_{H,l} e_{H,\tau} (h_{\bar{H}} - M_h^2)] + \\
& 2b_H [(e_{H,l} g_{H,\tau} + e_{H,\tau} g_{H,l}) (h_{\bar{H}} - M_h^2) + (d_{H,l} f_{H,\tau} + d_{H,\tau} f_{H,l}) (j_{\bar{H}} - M_h^2) - \\
& i_H (e_{H,\tau} f_{H,l} + e_{H,l} f_{H,\tau} + d_{H,l} g_{H,\tau} + d_{H,\tau} g_{H,l})] \quad . \quad (35)
\end{aligned}$$

The new coefficients in these expressions are

$$\begin{aligned}
a_H &= m_1^2 - \frac{1}{8} (g'^2 + g_2^2) (v_2^2 - 3v_1^2) & b_H &= -B\mu - \frac{1}{4} (g'^2 + g_2^2) v_1 v_2 \\
c_H &= m_2^2 + \frac{1}{8} (g'^2 + g_2^2) (3v_2^2 - v_1^2) & i_H &= -i_{\bar{H}}
\end{aligned} \quad (36)$$

and the derivatives of $\{e_H, d_H, f_H, g_H\}$ are listed in ref. [2]. For completeness, they are

$$\begin{aligned}
d_{H,l} &= \frac{1}{4} (4\lambda_\tau^2 + g'^2 - g_2^2) v_1 & d_{H,\tau} &= \frac{\lambda_\tau}{\sqrt{2}} A_\tau & e_{H,l} &= \frac{\lambda_\tau}{\sqrt{2}} A_\tau \\
e_{H,\tau} &= \frac{1}{2} (2\lambda_\tau^2 - g'^2) v_1 & f_{H,l} &= -\frac{1}{4} (g'^2 - g_2^2) v_2 & f_{H,\tau} &= -\frac{\lambda_\tau}{\sqrt{2}} \mu \\
g_{H,l} &= -\frac{\lambda_\tau}{\sqrt{2}} \mu & g_{H,\tau} &= \frac{1}{2} g'^2 v_2 \quad .
\end{aligned} \quad (37)$$

If we perform the substitution $M_{\bar{H}}^2 \leftrightarrow M_h^2$ in equations (33)-(35) we obtain the second derivatives of the mass of the heaviest scalar Higgs. Finally, for the staus we have

$$\begin{aligned}
M_{\tilde{\tau}_1, ll}^2 &= \frac{(g'^2 + g_2^2) (j_{\bar{\tau}_1} - M_{\tilde{\tau}_1}^2) + (2\lambda_\tau^2 - g'^2) (h_{\bar{\tau}_1} - M_{\tilde{\tau}_1}^2)}{M_{\tilde{\tau}_2}^2 - M_{\tilde{\tau}_1}^2} + \frac{A_{ll}(M_{\tilde{\tau}_1}^2)}{\bar{D}_{\tilde{\tau}}(M_{\tilde{\tau}_1}^2, M_{\tilde{\tau}_2}^2)} + \\
& \frac{B_{ll}(M_{\tilde{\tau}_1}^2)}{\bar{D}_{\tilde{\tau}}(M_{\tilde{\tau}_1}^2, M_{\tilde{\tau}_2}^2)} \\
M_{\tilde{\tau}_1, \tau\tau}^2 &= \frac{(2\lambda_\tau^2 - g'^2) (j_{\bar{\tau}_1} - M_{\tilde{\tau}_1}^2) + 4g'^2 (h_{\bar{\tau}_1} - M_{\tilde{\tau}_1}^2)}{M_{\tilde{\tau}_2}^2 - M_{\tilde{\tau}_1}^2} + \frac{A_{\tau\tau}(M_{\tilde{\tau}_1}^2)}{\bar{D}_{\tilde{\tau}}(M_{\tilde{\tau}_1}^2, M_{\tilde{\tau}_2}^2)} + \\
& \frac{B_{\tau\tau}(M_{\tilde{\tau}_1}^2)}{\bar{D}_{\tilde{\tau}}(M_{\tilde{\tau}_1}^2, M_{\tilde{\tau}_2}^2)} \\
M_{\tilde{\tau}_1, l\tau}^2 &= \frac{(g'^2 - 2\lambda_\tau^2) i_H}{M_{\tilde{\tau}_2}^2 - M_{\tilde{\tau}_1}^2} + \frac{A_{l\tau}(M_{\tilde{\tau}_1}^2)}{\bar{D}_{\tilde{\tau}}(M_{\tilde{\tau}_1}^2, M_{\tilde{\tau}_2}^2)} + \frac{B_{l\tau}(M_{\tilde{\tau}_1}^2)}{\bar{D}_{\tilde{\tau}}(M_{\tilde{\tau}_1}^2, M_{\tilde{\tau}_2}^2)} \quad , \quad (38)
\end{aligned}$$

with

$$\begin{aligned}
\bar{D}_{\tilde{\tau}}(M_{\tilde{\tau}_1}^2, M_{\tilde{\tau}_2}^2) &= (M_{\tilde{\tau}_2}^2 - M_{\tilde{\tau}_1}^2) [(a_{\bar{H}} - M_{\tilde{\tau}_1}^2) (c_{\bar{H}} - M_{\tilde{\tau}_1}^2) - b_{\bar{H}}^2] \\
D_{\tilde{\tau}}(M_{\tilde{\tau}_1}^2, M_{\tilde{\tau}_2}^2) &= (M_{\tilde{\tau}_2}^2 - M_{\tilde{\tau}_1}^2) [(a_H - M_{\tilde{\tau}_1}^2) (c_H - M_{\tilde{\tau}_1}^2) - b_H^2] \quad . \quad (39)
\end{aligned}$$

Again, the expressions for $M_{\tilde{\tau}_2}^2$ are obtained from these with a simple replacement. Because supersymmetry is softly broken the supertrace of the squared masses must be field-independent, so we can verify these formulae by checking that $Str \partial M^2 / \partial x \partial y = 0$.

3 Numerical results and discussion

We now apply our results for the one-loop tau masses to a vast MSSM parameter space. In order to try to take into account the effects of the particles' mass thresholds in the renormalisation running of the theory's parameters, we follow the procedure outlined in refs. [7] and use as input parameters $\alpha_1 = 0.01667$, $\alpha_2 = 0.032$, $\alpha_S = 0.1$, $m_b = 2.95$ GeV, $m_\tau = 1.75$ GeV and $m_t = 167.2$ GeV, at the scale M_Z . Accordingly we take the DRED value for $v^2 = v_1^2 + v_2^2 = (250.75 \text{ GeV})^2$, and use the supersymmetric two-loop β -functions to evolve all parameters between M_Z and the gauge unification scale M_U , defined as the point where the couplings α_1 and α_2 meet. At M_U we input the values of the soft parameters and determine μ and B by minimising the one-loop MSSM potential at the scale M , defined as the maximum of M_Z and the input scalar and gaugino masses. For the soft parameters our strategy was to choose a random value M_G , m_G and A_G and let the gaugino and scalar masses and A parameters vary randomly within a 30% interval of those central values, thus obtaining over 15000 points with input soft masses roughly in the interval $[10, 1000]$ GeV and $-4 < A_G < 4$ TeV. Further, we have taken $2.5 \leq \tan \beta \leq 6.5$ and considered both possible signs for the μ parameter. We then impose experimental bounds on the sparticles' masses from ref. [8], except, obviously, the bounds on the tau masses, as we are interested in checking whether the one-loop contributions change their values considerably. As it turns out, for our choice of parameter space, after all other experimental cuts have been applied the remaining points (over 10000 of them) correspond to tau masses above the current experimental bound (81 GeV) for all but a handful of points. The results of this "scan" of the MSSM can be seen in figures (1)- (3). In fig. (1) we plot the mass difference between the one-loop and tree-level masses for the lightest tau, against the maximum M of the input soft masses and $M_Z - M$ is of the order of the largest masses present in ΔV_1 and as such should constitute a good choice for renormalisation scale. Several observations about this plot: for S smaller than about 200 GeV there is no substantial difference between the tree-level and one-loop results. For larger values, though, there are sizeable differences, usually smaller than 10 GeV. We observe that for the majority of points in the chosen parameter space the mass of the lightest tau increases. As expected, the one-loop contributions are small (typically less than 5%) but we find they are not negligible. Figure (2) is the analogous of the previous one, but looking at the one-loop/tree-level mass difference for the heaviest tau - we see the one-loop contributions tend to decrease the mass of the heavier tau, by as much as ~ 20 GeV. Again, these contributions are only a few percent of the total mass (typically less than 8%) but not at all insignificant. Now, since the lightest CP-even higgs boson is likely to be discovered before the taus, it is useful to look at the relationship between $\Delta M_{\tilde{\tau}}$ and M_h (we use the full one-loop higgs mass). In fig. (3) we plot these two quantities one against the other - despite the fact that the largest values of $\Delta M_{\tilde{\tau}}$ occur (naturally) for higher values of the input soft masses, they do not necessarily correspond to large values of M_h .

In conclusion, we computed the one-loop contributions to the tau masses in the effective potential approach and showed they are usually quite small, but nevertheless sizeable. In particular, the lighter tau mass is shown to increase for the majority of input values we considered. If the taus are ever discovered and their mass measured accurately, these mass differences could be instrumental in narrowing the parameter space of the MSSM. Caution must be exercised in reading these results, though: the e.p.a., we repeat, is an approximation to the real mass. For instance, as discussed in ref. [4], the resulting mass has a small renormalisation scale dependence. We confirmed that fact by changing the value of M , doubling it or reducing it to 100 GeV, but the resulting changes in $\Delta M_{\tilde{\tau}}$ are indeed very small. We also recall that the e.p.a. results for the Higgs sector were off by a few GeV, but again from ref. [4] we observe the e.p.a. tends to *underestimate* the real masses. As such, our conclusion regarding the increase of the lightest tau mass should hold. Of course, the validity of the e.p.a. can only be established by perform-

ing the full diagrammatic calculation, and perhaps the fact the e.p.a. is predicting measurable differences for $M_{\tilde{\tau}}$ is sufficient reason to undertake it. We also observe that an e.p.a. calculation in the stop sector should yield larger one-loop contributions for the simple reason it produces second derivatives of the masses proportional to λ_t^2 . This work is now under preparation.

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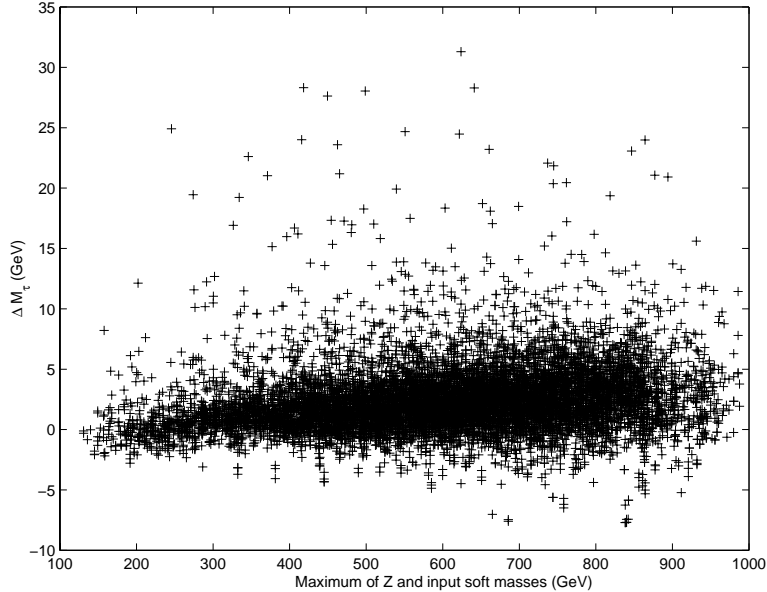


Figure 1: The maximum M of M_Z and the input soft masses versus the mass difference between the one-loop and tree-level calculated lightest stau mass, $\Delta M_{\tilde{\tau}} = M_{\tilde{\tau}}^{1-loop} - M_{\tilde{\tau}}^{tree}$.

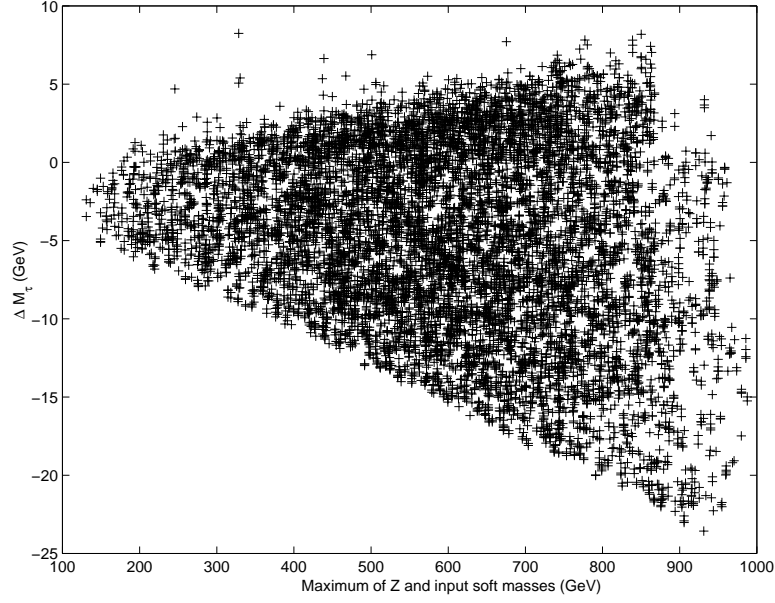


Figure 2: The maximum M of M_Z and the input soft masses versus the mass difference between the one-loop and tree-level calculated heaviest stau mass.

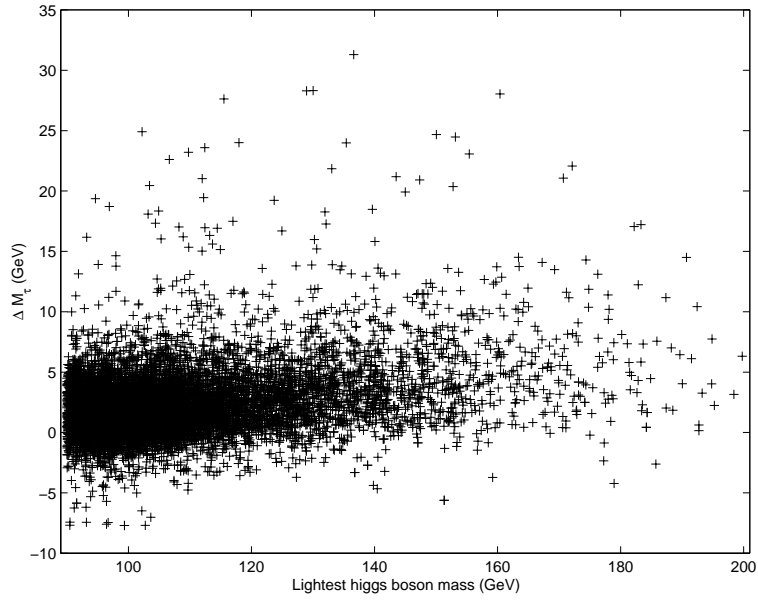


Figure 3: Lightest CP-even higgs boson mass (one-loop) versus the mass difference between the one-loop and tree-level calculated lightest stau mass.